

題目：設 x 為實數， $f(x) = 3x^2 + 3x + \frac{27}{x^2 + x + 2}$ ，

則 (1) $f(x)$ 的最小值 = _____ (2) 又此時 $x =$ _____

答案：(1)對於任意實數 x ， $x^2 + x + 2 > 0$ 恆成立 ($\because D < 0$)

由算幾不等式知，

$$\begin{aligned} f(x) &= 3x^2 + 3x + \frac{27}{x^2 + x + 2} = 3(x^2 + x + 2) + \frac{27}{x^2 + x + 2} - 6 \\ &\geq 2\sqrt{3(x^2 + x + 2)\left(\frac{27}{x^2 + x + 2}\right)} - 6 = 18 - 6 = 12 \end{aligned}$$

$\therefore f(x)$ 的最小值 = 12

(2)由算幾不等式知，產生最小值時

$$3(x^2 + x + 2) = \frac{27}{x^2 + x + 2}$$

$$\therefore (x^2 + x + 2)^2 = 9 \Rightarrow x^2 + x + 2 = \pm 3 \quad (\text{負不合})$$

$$\therefore x^2 + x + 2 = 3 \Rightarrow x^2 + x - 1 = 0$$

$$\therefore x = \frac{-1 \pm \sqrt{5}}{2}$$